On crack growth path of rock materials subjected to tensile loading

M.R. Ayatollahi *, M.R.M. Aliha , M. Rezaei

Corresponding author: Fatigue and Fracture Lab., Department of Mechanical Engineering, Iran University of Science and Technology, Narmak, Tehran, 16846, Iran
E-mail address: m.ayat@iust.ac.ir

Abstract
Underground mining and tunneling have been developed extensively in many geological related applications like underground railways, gas and oil wellbores and deep tunnel mining. Since the tunneling and excavation are based on a process of crack growth in the rock masses, the investigation of crack growth behavior and controlling its propagation path is an important task for designing and constructing the underground structures. In this paper, the stability of crack growth path under tensile mode of loading in a rock sample is studied both experimentally and theoretically using fracture mechanics approach. A series of fracture experiments were conducted on the Iranian Harsin white marble using the double cantilever beam (DCB) and semi circular bend (SCB) configurations. While the path of crack growth in the SCB specimens was stable and self similar, the crack curving was observed for the DCB specimens. The directions of crack growth initiation for the DCB specimen did not comply with the predictions of the available classical fracture criteria. However, it is shown that a generalized fracture criterion which takes into account the effects of both specimen geometry and material type can predict very well the fracture behavior of the tested rock material. It is shown that the crack curving in the tested DCB specimens made of coarse grain Harsin marble is mainly due to the noticeable positive T-stresses that exist in the DCB specimens and the relatively large size of fracture process zone in this material.

1. Introduction
Nowadays, underground mining and tunneling have been developed extensively in many geological related applications like underground railways, gas and oil wellbores and deep tunnel mining. In such applications, the higher safety and stability of the constructed tunnels, the faster speed of excavation and rock cutting. Increasing the productivity of oil and gas wells is also one of the main designing parameters for the geology and mining engineers. Since the excavation of rock masses is basically a process of crack growth in the rocks, the investigation of crack growth behavior and controlling its propagation path, is an important task for designing and constructing the underground structures. The existence of initial cracks, flaws, joints, beddings and weak planes and inherent discontinuities is one of the main natural characteristics of rock masses. Hence, for a more precise analysis and design of the rock structures, the influence of cracks and flaws should be studied. The linear elastic fracture mechanics (LEFM) can provide a suitable framework to study the crack growth behavior in cracked rock masses. LEFM deals with various experimental methods and theoretical fracture criteria for evaluating the onset of crack growth and the direction of fracture path. In this paper, the stability of crack growth path under tensile mode of loading in a rock sample (Iranian Harsin white marble) is studied both experimentally and theoretically using two test samples namely: the double cantilever beam (DCB) and the semi circular bend (SCB) configurations. Also, the parameters affecting the initiation of fracture and the path of the crack extension in the tested rock, is investigated theoretically. Using a generalized fracture

criterion, it is shown that the stability of mode I crack growth for the tested rock material, is controlled by the sign and the magnitude of T-stress and the size of fracture process zone as well.

2. Theoretical aspects

Crack growth in rocks or other geo-materials often occurs under crack-opening deformation (known as mode I) and in a brittle fracture manner. Since the geometry and loading conditions in mode I deformation are symmetric with respect to the crack plane, it is usually expected that the mode I crack growth would be self-similar i.e. along the direction of original crack. This phenomenon is in agreement with the predictions of the classical fracture criteria for mode I loading [1-3]. These criteria suggest that the angle of fracture initiation in pure mode I loading, is always zero and the path of crack growth is stable and along the initial crack line. However, a review of literature shows that some of the mode I fracture paths observed for various engineering materials like rocks are not along the original crack line. Indeed, the mode I fracture may grow in some cases along a curvilinear path. Thus, some researchers have attempted to investigate the stability of crack growth path under mode I loading. For example Cottrell and Rice [4], used a well known maximum tangential stress (MTS) criterion for investigating the mode I crack curving problem. The elastic tangential stress for mode I loading condition can be written as an infinite series expansion [5]:

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} K_{\mathrm{I}} \cos^3 \frac{\theta}{2} + T \sin^2 \theta + O(r^{\frac{1}{2}}) \qquad (1)$$

where r and θ are the polar co-ordinates, K_{I} is the mode I stress intensity factor and T is a constant and nonsingular stress term usually called the T-stress. The higher order terms $O(r^{1/2})$ are negligible near the crack tip. According to Cottrell and Rice [4], the sign of T-stress is the main controlling parameter for the stability of crack growth under mode I loading. Based on their study, the path of crack growth for those mode I specimens having a negative T-stress is stable and along the original crack line and conversely for those specimens that the sign of T-stress is positive, the crack curving would be occurred. Although their theory could provide good predictions for the stability of crack growth in some specimens, their criterion failed for some other mode I experiments. Then Ayatollahi and his co-workers [6], presented a more accurate criterion for justifying the mode I crack growth behavior. Based on Ayatollahi et al. [6], the direction of fracture initiation angle in mode I depends not only on the sign of T-stress but also on the magnitude of T-stress and the material properties as well. Brittle fracture occurs when the maximum tangential stress ($\sigma_{\theta\theta}$) at a critical distance r_c in front of the crack tip, reaches a critical value ($\sigma_{\theta\theta c}$). The required conditions for satisfying the above statement are:

$$\frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0 \quad , \quad \frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2} \prec 0 \qquad (2)$$

Consequently Ayatollahi et al. [6], showed that the direction of fracture initiation angle is zero if the value of $B.$▯ < 0.375. For $B.$▯ > 0.375, the direction of fracture initiation (θ_m) is calculated from:

$$\theta_m = \pm 2\cos^{-1}\left[\frac{3}{32B\alpha} + \sqrt{\left(\frac{3}{32B\alpha}\right)^2 + \frac{1}{2}}\right] \tag{3}$$

where B is the biaxiality ratio and α is a parameter related to the size of critical distance (r_c). These parameters are written as:

$$B = \frac{T\sqrt{\pi a}}{K_I} \quad , \ \alpha = \sqrt{\frac{2r_c}{a}} \qquad \Rightarrow \ B.\alpha = \frac{T}{K_I}\sqrt{2\pi r_c} \tag{4}$$

For investigating the validity of the mentioned criteria for the stability of mode I crack in rocks, a series of fracture specimens were tested using a sample rock material and by means of two different mode I geometries. In the next section, the experimental results are presented.

3. Fracture experiments on Harsin marble

A schematic representation of the test specimens used for mode I fracture experiments are shown in Fig. 1. These specimens are: the semi circular specimen containing an edge crack and subjected to three-point bend loading (SCB) specimen and the double cantilever beam (DCB) specimen subjected to tensile pin loading. For conducting the fracture experiments, several SCB and DCB specimens were manufactured from an Iranian white marble rock with coarse grains (Harsin marble). The thickness (t) of both test samples was approximately 20 mm. The crack length (a) was also considered as a variable in both SCB and DCB specimens. The specimens were then loaded by means of suitable fixtures using a ball screw testing machine until the final fracture. It was observed from the experiments that, the fracture paths of all the SCB specimens were along the original crack line but the fracture paths for the tested DCB specimens were deviated from the crack line. The fracture paths for some of the broken SCB and DCB specimens are shown in Fig. 2.

4. Results and discussion

For investigating the crack growth stability of the tested rock samples, based on the fracture criteria [4,6] mentioned in the previous section, the value of T-stress (or B) should be known. The fracture parameters of K_I and T in both DCB and SCB specimens at the onset of fracture were calculated by means of the finite element code ABAQUS and using the fracture load obtained for each specimen. The value of r_c in rock materials is usually estimated by the size of fracture process zone in front of the crack tip. Schmidt [7] has proposed the following relation for evaluating the size of fracture process zone (FPZ) in rock materials under mode I loading:

$$\text{size of FPZ } (r_c) = \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_t}\right)^2 \tag{5}$$

where K_{Ic} and σ_t are the pure mode I fracture toughness and the tensile strength of rock, respectively. Using the ISRM suggested methods [8,9], the values of K_{Ic} and σ_t was obtained as 1.50 MPa $m^{0.5}$ and 6.01 MPa from experiments conducted on appropriate test specimens. By replacing the obtained values of K_{Ic} and σ_t into Eq. (5), the process zone size r_c was found about 9.91 mm. it should be noted that the size of r_c in most of rock materials is considerably greater than the size of r_c for many other engineering materials like metals, polymers and ceramics. Then by using the corresponding values of $B.\alpha$ for any test samples, the direction of fracture initiation angle can be determined theoretically by the maximum tangential stress criterion suggested by Ayatollahi et al. [6]. The obtained test results including the fracture angles experimentally measured from the broken samples and together with the predicted fracture angles are presented in Table 1 for both SCB and DCB specimens tested with different geometry and loading conditions.

It is seen from this Table that the *T*-stress in all of the tested DCB specimens is noticeably positive and the corresponding $B.\alpha$ values are greater than 0.375. Hence, according to both criteria i.e. Cottrell and Rice [4] and also Ayatollahi et al. [6], the crack curving should occur. These predictions are in agreement with the fracture pattern observed from the broken samples (see Fig. 2). However, according to Table 1 while the *T*-stress for some of the tested SCB specimens was positive, the crack growth paths for all of the tested SCB specimens were stable and along the original crack line (i.e. θ_m was equal to zero). This result is not consistent with the theory suggested by Cottrell and Rice [4]. But the criterion suggested by Ayatollahi et al. [6] was in agreement with the fracture path observed for the SCB specimens as well. According to Ayatollahi et al. [6] the crack initiation angle for all of the tested SCB specimens would be zero because the corresponding values of $B.\alpha$ for any SCB specimens studied in this research were less than 0.375. Consequently, the SCB specimens should have stable crack growth path when they are subjected to pure mode I loading. Therefore, the generalized criterion suggested in [6] which takes into account the effects of magnitude and sign of *T*-stress and also the type of material (which was related to the size of fracture process zone) can provide more reliable estimates for crack growth stability in mode I cracks in rock masses.

5. Conclusions

- Crack growth stability in mode I cracks were studied both experimentally and theoretically for a rock material using DCB and SCB specimens.
- While the path of crack growth path in the SCB specimens were stable and along the original crack line, crack curving was observed for the DCB specimens.
- The very high positive *T*-stress that exists in the DCB specimen and the relatively large size of fracture process zone in the tested marble rock were the main reasons for the crack growth curving in the DCB specimen.

6. References

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Table 1: Experimental and theoretical results obtained for the direction of fracture initiation angle in the tested rock samples.

specimens			experimental fracture angle (Deg)	T- stress (MPa)	$B\alpha$	theoretical fracture angle (Deg)
SCB	$a/R = 0.45$	$S/R = 0.50$	0	-2.0750	-0.3982	0
			0	-2.3180	-0.3982	0
	$a/R = 0.50$	$S/R = 0.50$	0	-1.0277	-0.1554	0
			5	-1.3086	-0.1554	0
	$a/R = 0.66$	$S/R = 0.60$	0	0.3249	0.0659	0
			10	0.5796	0.0659	0
DCB	$a/w = 0.20$		60	6.1099	1.6509	79.98
			55	6.7400	1.6515	79.98
	$a/w = 0.30$		70	7.7866	1.8258	81.02
			59	9.0032	1.8259	81.02
	$a/w = 0.70$		72	8.3901	2.1228	82.36
			70	9.0026	2.1229	82.36

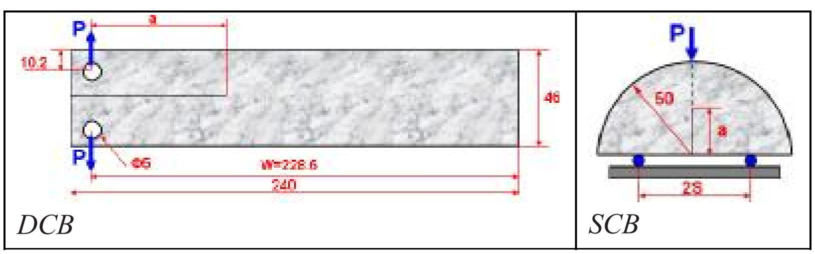

Fig. 1: Geometry and loading conditions of SCB and DCB specimens subjected to mode I loading (dimensions: mm).

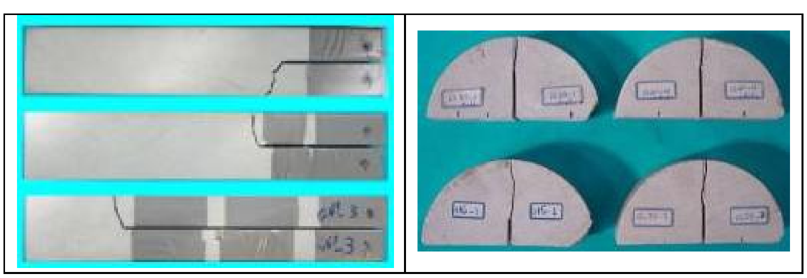

Fig. 2: fracture paths observed for some of the tested SCB and DCB specimens tested with Harsin marble.